Section 4

Mathematical Reasoning: Aspects of Cognition and Practice
Organizer: Marianna Antonutti Marfori (MCMP)
Participants: Valeria Giardino (CNRS/Université de Lorraine), Paula Quinon (University of Lund), Colin Jacob Rittberg (Vrije Universiteit Brussel), Fenner Tanswell (University of Oxford)

Abstract:

This symposium aims at assessing the progress made in the philosophy of mathematical practice and outline avenues for further research by presenting and discussing some representative and very recent advances in the area. The invited speakers will bring in tools from other areas of philosophy—such as aesthetics and virtue epistemology, as well as from other disciplines such as psychology and cognitive neuroscience—in order to address central questions about mathematical reasoning and how we obtain knowledge in everyday mathematical practice. (3) Recent semantic and syntactic construals of theoretical equivalence.

Titles and Abstracts of the talks

1.
Valeria Giardino: Manipulative Imagination: How to move things around in mathematics
In my talk, I will first introduce the notion of manipulative imagination as particularly salient in mathematics. Such a form of imagination is central to many areas of topology, for example knot theory (De Toffoli & Giardino, 2014), low-dimensional topology (De Toffoli & Giardino, 2015) and braid theory (De Toffoli & Giardino, 2016). Second, I will discuss a possible characterization of manipulative imagination as based on the notion of imagination as "make-believe" (Walton, 1990) and on that of "affordance" (Gibson, 1979).

2.
Paula Quinon: Conceptual Spaces Model of Early Number Cognition
(Gemel-Quinon 2014) proposed a conceptual spaces model of pre-verbal representations resulting from processing quantities by the Approximate Number System (ANS). This talk extends this model to other cognitive systems participating in the number concept creation. I will concentrate on the conceptual content that numerical expressions get by subitizing through the Parallel Individuation System. Cognitive scientists observed that children learn number-word meanings one at a time and in order; children are thus divided into number-knower-levels based on their understanding of number-word meanings. I will argue that this model accounts for conceptual change between the levels, and explain its relation to the ANS model.

3.
Colin Jacob Rittberg and Fenner Tanswell: Proofs Gone Wrong

To find out about what makes for good reasoning in mathematics it is helpful to investigate what is wrong with bad mathematical reasoning. In this talk, we draw out various ways in

which mathematical reasoning can be defective. Our focus is on mathematical proofs. Proofs cannot only be logically flawed, they can be inappropriate for the audience or context, boring, sloppy, make use of too much “heavy machinery” of mathematics, etc. Building on our previous work of bringing virtue-theory to the philosophy of mathematics, we investigate cases of flawed mathematical reasoning to get a grip on intellectual vices in mathematics.

4.
Marianna Antonutti Marfori: De Re Knowledge of Mathematical Truths: An algebraic case study
This talk will examine two theorems: that every countable commutative ring has a maximal ideal, and that every countable commutative ring has a prime ideal. Every maximal ideal is a prime ideal, but results from Reverse Mathematics show that the two theorems reverse to two subsystems of second-order arithmetic of different strengths; this fact will be used to make more precise the idea that the proof in the stronger system gives us additional knowledge with respect to the proof in the weaker system. This qualitative difference will be analysed in terms of the distinction between de re and de dicto knowledge.